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Micromagnetic understanding of current-driven domain wall motion in patterned nanowires

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Abstract. – In order to explain recent experiments reporting a motion of magnetic domain walls (DW) in nanowires carrying a current, we propose a modification of the spin transfer torque term in the Landau-Lifchitz-Gilbert equation. We show that it explains, with reasonable parameters, the measured DW velocities as well as the variation of DW propagation field under current. We also introduce coercivity by considering rough wires. This leads to a finite DW propagation field and finite threshold current for DW propagation, hence we conclude that threshold currents are extrinsic. Some possible models that support this new term are discussed.

Recent research on magnetic nanostructures has shown that effects caused by an electric current flowing across a nanostructure may dominate over the effects due to the field generated by the same current [1,2]. Most of the work up to now, experimental and theoretical, has been devoted to the 3-layer geometry (2 magnetic layers separated by a normal metal spacer, with lateral dimensions well below the micrometer so as to get single domain behaviour). Under current, generation of spin waves [3], layer switching [2] and precession of magnetization [4] have been observed. All these results could be qualitatively explained by the spin transfer model [1]. On the other hand, the situation with an infinite number of layers, namely a magnetic nanowire containing a magnetic domain wall (DW), has just started to be studied. Here, under the sole action of a current, the DW may be moved along the wire, as confirmed by several experiments [5–10]. The situation is however more complex than with 3 layers, as the evolution of the current spin polarization across the DW has to be described, so that a deep connection with the problem of DW magnetoresistance exists. Two limits have been identified by the theories, namely the thin and thick DW cases. The length to which the DW width has to be compared is, depending on the model, the spin diffusion length [11], the Larmor precession length [12], or the Fermi wavelength [13]. These lengths are below or of the order of a nanometer in usual ferromagnetic metals. In the experiments cited above (nanowires of typical width 100 nm and thickness 10 nm), the DW width is of the order of 100 nm, much above these lengths. Thus, the approximations of current polarization adiabaticity and full transfer of angular momentum to the local magnetization apply, and the Landau-Lifchitz-Gilbert (LLG) equation of magnetization dynamics becomes

$$\dot{\vec{m}} = \gamma_0 \vec{H} \times \vec{m} + \alpha \vec{m} \times \dot{\vec{m}} - \left(\vec{u} \cdot \vec{\nabla}\right) \vec{m}. \tag{1}$$

The symbols are: $\vec{m}$ unit vector along the local magnetization, γ_0 gyromagnetic constant, $\vec{H}$ micromagnetic effective field, M_s spontaneous magnetization and α Gilbert damping constant. The velocity $\vec{u}$ is a vector directed along the direction of electron motion, with an amplitude

$$u = JPg\mu_\mathrm{B}/\left(2eM_\mathrm{s}\right), \tag{2}$$

where J is the current density and P its polarization rate (u is positive for $P > 0$, *i.e.* for carriers polarized along the majority spin direction). For permalloy the factor $g\mu_\mathrm{B}/(2eM_\mathrm{s})$ amounts to $7 \times 10^{-11}\,\mathrm{m}^3/\mathrm{C}$. The local form (1), already introduced by several authors [14–17], can be seen as the continuous limit of the Slonczewski spin transfer term between infinitely thin successive cross-sections of the nanowire.

The consequences of (1) on the dynamics of a transverse wall (TW —fig. 1a) have been well studied qualitatively [13, 18]. Using the integrated equations for the DW dynamics [19, 20], it was found that 2 regimes exist. At low current, the spin transfer torque is balanced by an internal restoring torque. The DW magnetization tilts out of the easy plane (as first explained by Berger [21]), but the DW does not move steadily under current. Above threshold, the internal torque is not sufficient and DW motion occurs, together with a continuous precession of the DW magnetization (in fig. 1: $a \to d \to \bar{a} \to \bar{d} \ldots$, the bar meaning π rotation around the x-axis). The threshold is linked to the steepness of the potential well that defines the equilibrium orientation of the TW magnetization (mainly a magnetostatic effect, see fig. 1).

These qualitative findings were fully confirmed by micromagnetic computations [22] uncovering, however, a big discrepancy with experimental results. The intrinsic current thresholds computed for steady propagation of a TW in a perfect wire were found to be more than 10 times larger than experimental values. Thermal fluctuations cannot change the situation because the energy barrier against DW magnetization rotation proves very large (for a $120 \times 5\,\mathrm{nm}^2$ permalloy wire, the energy difference between states (d) and (a) in fig. 1 is 230 $k_\mathrm{B}T_\mathrm{amb}$). Therefore, we need to introduce corrections to perfect adiabaticity and pure local spin transfer in the micromagnetic models. In this letter, we show how the inclusion of a second term describing the current torque, as introduced previously in the 3-layer geometry [23], can resolve this discrepancy. Our approach is twofold: first, we study how micromagnetic

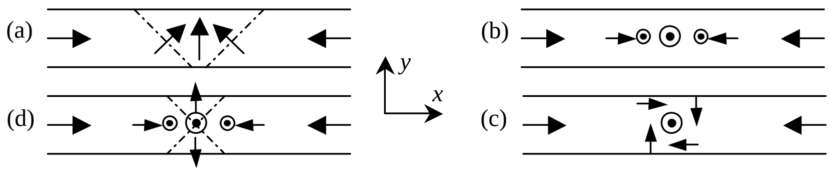

Fig. 1 – Schematic domain wall structures in a soft nanowire. They are governed by magnetostatics and depend on the wire cross-section: (a) transverse wall, (b) perpendicular transverse wall, (c) vortex wall. As the fabricated wires are very wide compared to the exchange length (5 nm for permalloy), the energy maximum situation (b) decays to a wall with an antivortex (d) [24]. In cases (a, b) the DW magnetization angle ϕ with respect to the y-axis is 0 and 90°, respectively. The stable structures are (a) at smaller and (c) at larger cross-sections [25].

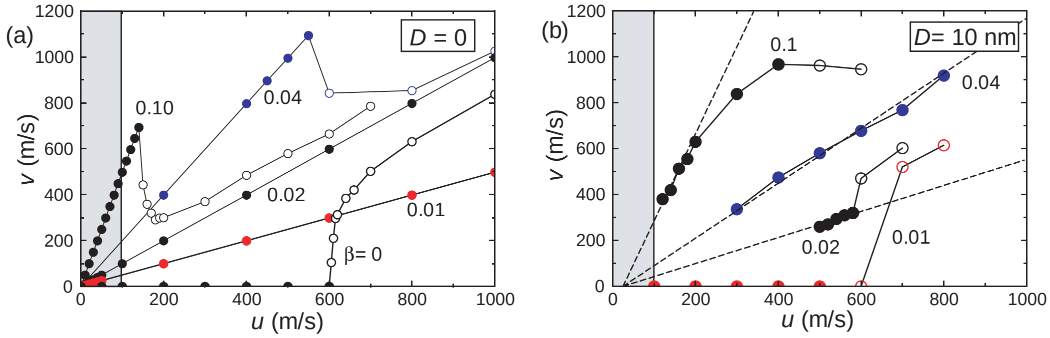

Fig. 2 – Steady velocity computed for a transverse domain wall by micromagnetics in a $120 \times 5\,\mathrm{nm}^2$ wire as a function of the velocity u representing the spin-polarized current density (2), with the relative weight (β) of the exchange field term as a parameter. Open symbols denote vortices nucleation. The shaded area indicates the available experimental range for u. (a) Perfect wire and (b) wire with rough edges (mean grain size $D = 10\,\mathrm{nm}$). The dashed lines display a fitted linear relation with a $25\,\mathrm{m/s}$ offset.

results are modified when this term is phenomenologically included. In a second step, we examine several models of the current effect that may justify the existence of such a term, and try to estimate its magnitude.

The modified LLG equation, again obtained in the continuous limit, now reads

$$\dot{\vec{m}} = \gamma_0 \vec{H} \times \vec{m} + \alpha \vec{m} \times \dot{\vec{m}} - \left(\vec{u} \cdot \vec{\nabla}\right) \vec{m} + \beta \vec{m} \times \left[\left(\vec{u} \cdot \vec{\nabla}\right) \vec{m}\right]. \tag{3}$$

As discussed later, the non-dimensional parameter β is much smaller than unity, hence comparable to α. The solved form of (3) shows that the new term modifies the initial spin transfer torque by a second-order quantity, and directly competes with the damping term associated to the first term. This is very appealing, as one sees directly from (1) that for zero damping the solution under current is just the solution at zero current, but moving at velocity $\vec{u}$.

In order to test the influence of this new term, micromagnetic numerical simulations were performed. Similarly to previous work [24], a moving calculation region centered on the DW represented an infinite wire. Wire width and thickness corresponded to experiments [7,8], and material parameters were those typical of permalloy ($M_\mathrm{s} = 8 \times 10^5\,\mathrm{A/m}$, exchange constant $A = 10^{-11}\,\mathrm{J/m}$ and no anisotropy) with, in most cases, damping fixed to $\alpha = 0.02$ [24]. The mesh size was $4 \times 4 \times 5\,\mathrm{nm}^3$. It was checked first that the Oersted field generated by the current had a negligible effect for current values corresponding to experiments. Indeed, because of the small thickness of the sample, this field is essentially perpendicular and localized at sample edges. Its maximum value for an extreme case $u = 100\,\mathrm{m/s}$, $P = 0.4$ in a $120 \times 5\,\mathrm{nm}^2$ wire is $\mu_0 H_z = 0.017\,\mathrm{T}$, a value well below the perpendicular demagnetizing field (1 T), that results in a very small out-of-plane magnetization rotation. Consequently, all results shown below were computed without the Oersted field. Transverse, but also vortex walls, were considered.

Figure 2a shows the wall velocity v under zero field as a function of the velocity u (proportional to the current) with β as a parameter. The sample is 120 nm wide and 5 nm thick [7], with a DW of the transverse type (fig. 1a). The $\beta = 0$ curve displays an absence of DW motion for $u < u_\mathrm{c} = 600\,\mathrm{m/s}$, and a rapid increase of v towards u above the threshold, as inferred previously [13, 17, 22]. Curves with $\beta > 0$ differ markedly from this behaviour, as DW motion is obtained at any finite u (keep in mind that the wire is perfect). Velocity increases linearly with u and β, up to a breakpoint where it decreases (or increases if $\beta < \alpha$

—not shown). Results obtained for other α values indicate that all of them can be described as $v = \beta u/\alpha$ below the breakpoint. Above the breakpoint, the DW structure examination reveals the periodic injection of antivortices at wire edges, which cross the wire width and are expelled, similarly to the field-driven case [24] or to the current-driven case with $\beta = 0$ [22], above their respective thresholds.

The same behavior is obtained at low u for a VW structure in a larger wire (width 240 nm, thickness 10 nm). This surprising result (for field-driven propagation, wall mobility is much lower for a VW [25]) can be explained by the global arguments of Thiele [26] about steady-state motions. Indeed, eq. (3) can be transformed into a generalized Thiele relation,

$$\vec{F} + \vec{G} \times (\vec{v} - \vec{u}) + \overleftrightarrow{D}\,(\alpha\vec{v} - \beta\vec{u}) = \vec{0}, \tag{4}$$

with $\vec{F}$ the static force, $\vec{G}$ the gyrovector (along z here) and $\overleftrightarrow{D}$ the dissipation dyadic [26]. The solution is then $\vec{v} = \beta\vec{u}/\alpha$ as long as the restoring force keeping the vortex in the wire can balance the gyrotropic term. Otherwise, the VW transforms into a TW by lateral expulsion of the vortex.

Experiments on single-layer wires of permalloy [7, 8, 10] show however that a threshold current exists for DW propagation, similarly to field-driven propagation (for example, a propagation field $H_{\mathrm{P}} \sim 20$ Oe was measured in ref. [7]). This is not reproduced by our results for perfect wires. A roughness of the wire edges was therefore introduced, characterized by a mean grain size D (see ref. [24] for details). Figure 2b shows how the results of fig. 2a are modified for $D = 10$ nm, a value which results in an extrapolated propagation field $H_{\mathrm{P}} \sim 10$ Oe, with velocity *vs.* field curves given in ref. [22]. At low β (< 0.02), the DW is blocked for $u < u_{\mathrm{c}}$, showing that roughness-induced DW pinning dominates. Above u_{c}, the DW moves via nucleation of antivortices and vortices, again similarly to the perfect case with $\beta = 0$. If β is larger (0.02–0.1), a window of DW propagation without vortex nucleation exists (filled symbols), including values of u well below u_{c}. Larger β values result in DW motion at lower u. However, with our coercivity model and the absence of thermal fluctuations, continuous DW propagation at "low" velocities ($v < 300$ m/s) is not possible as the DW eventually stops at positions with a larger wire width fluctuation. This may be largely due to our coercivity model, as in the same conditions a VW moves already at $v \approx u = 100$ m/s, and for regular notches TW propagation at $v < 10$ m/s is obtained. By comparing with fig. 2a, it is clear that the rough wire data cannot be fitted by merely introducing an offset u_{P} into the results for a perfect wire, as done usually in the field-driven case. On the other hand, simple proportionality of v to u gives a very good fit. However, it would be as good if a small offset existed, as shown by the dashed lines in fig. 2b that assume $u_{\mathrm{P}} = 25$ m/s. Thus, this model of coercivity does create a propagation field H_{P} and is consistent with a threshold current u_{P} for DW propagation, even if the data do not allow to determine precisely its value. An important additional result of the simulations is that, for a rough wire, the velocity is reduced when compared to the perfect-case relation $v = (\beta/\alpha)u$. The reduction factor is not a constant, as, for example, at $D = 10$ nm, $\alpha = 0.02$ and $\beta = 0.04$, the factors for a TW are 0.6, 0.7 and 0.4 for 120×5, 240×5 and $240 \times 10\,\mathrm{nm}^2$ strips, respectively, whereas they are in the range 0.5–0.6 for a VW.

Experimental propagation threshold currents u_{P} [7, 8, 10] for permalloy strips amount to $P \times 50$ m/s (TW) and $P \times 80$ m/s (VW), for cross-sections 120×5, 240×10 and $200\times(5$–$35)\,\mathrm{nm}^2$, the current polarization P being unknown. These values are compatible with our results at $D = 10$ nm, which was chosen in order to reproduce typical propagation fields. Although the value of β cannot be determined at this stage, we infer from our results that it should be larger than 0.01. In addition, the measurement of the propagation field under current [7] showed a linear variation of H_{P} *vs.* u, with $\Delta H_{\mathrm{P}} = 3$ Oe for $\Delta u = P \times 100$ m/s. As for a perfect wire [27] one has $v = \mu H + \beta u/\alpha$, we evaluate $\Delta H_{\rm P}/\Delta u = \beta/(\alpha\mu)$. The mobility calculated by micromagnetics for this nanowire being $\mu = 26.5\,{\rm ms}^{-1}\,{\rm Oe}^{-1}$ [25], we estimate, for $\alpha = 0.02$ and $P = 0.4$, $\beta = 0.04$. A micromagnetic calculation for this situation, on the same $120 \times 5\,{\rm nm}^2$ wire with $\alpha = 0.02$, $\beta = 0.04$ and $u = \pm 50\,{\rm m/s}$, gives $\Delta H_{\rm P} \sim 7.8\,{\rm Oe}$ which fully agrees with the above ($0.4 \times 7.8 = 3.1$). Another experimental data is the DW propagation velocity v (only one measurement, ref. [8]). It was seen to increase from 2 to 6 m/s as the current velocity was increased from $P \times 80$ to $P \times 90$ m/s. The average slope of these data is consistent with, in a perfect wire, $\beta = 0.03$, $\alpha = 0.02$, and $P = 0.4$. Note that i) the β value should be larger because of the velocity reduction in a rough wire, and ii) the determination of β is subject to uncertainty, as the coercivity source we considered may not be the only meaningful one. To summarize, we see that the experimental results can be reproduced by introducing the β term in (3), with a value of about 0.04 for permalloy.

We describe now how the numerical micromagnetic results shown above can be qualitatively understood in the TW case, in the framework of the extended Slonczewski equations describing DW motion. These are constructed by assuming a given DW profile with some parameters (here the wall position q and magnetization angle ϕ —see fig. 1). In the case of pure spin transfer, the LLG equation could be derived from a least-action principle, allowing an easy derivation of the equations [22]. This is no longer true with the additional term and the original procedure must be applied [19, 20]. For a one-dimensional TW profile, characterized by a DW width parameter Δ, one then gets

$$\dot{\phi} + \alpha\dot{q}/\Delta = \gamma_0 H_{\rm a} + \beta u/\Delta, \qquad \dot{q}/\Delta - \alpha\dot{\phi} = \gamma_0 H_{\rm K} \sin\phi\cos\phi + u/\Delta, \tag{5}$$

with $\Delta = \Delta(\phi)$ the wall width parameter minimizing the DW energy at wall magnetization angle ϕ. In the simplest 1D model [22] one has $\Delta(\phi) = (A/(K_0 + K\sin^2\phi))^{1/2}$, with K_0 the effective longitudinal anistropy and K the transverse one ($H_{\rm K} = 2K/(\mu_0 M_{\rm s})$). One sees from (5) that the additional term enters the equations similarly to an applied field $H_{\rm equiv} = u\beta/(\gamma_0\Delta)$,

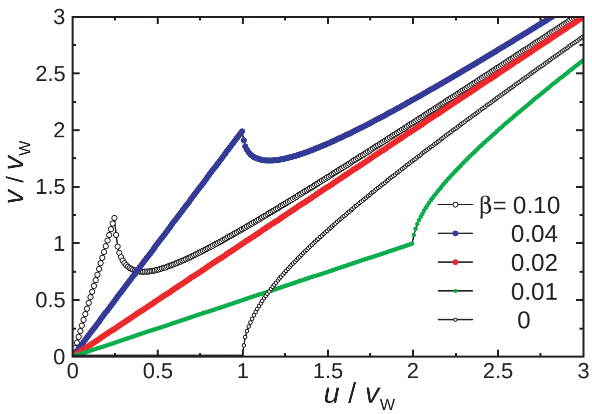

Fig. 3 – Domain wall propagation velocity under current as obtained from the one-dimensional model, assuming a perfect wire and a constant wall width parameter Δ, for a damping constant $\alpha = 0.02$. The curves' shape is extremely similar to the numerical results (fig. 2a). Velocities are normalized to the Walker threshold $v_{\rm W} = \gamma_0\Delta H_{\rm K}/2$.

explaining DW motion at any u in a perfect wire. At zero applied field one gets

$$\dot{\phi} = \frac{\alpha}{1+\alpha^2}\left[\frac{\beta-\alpha}{\alpha}\frac{u}{\Delta} - \frac{\gamma_0 H_{\rm K}}{2}\sin 2\phi\right], \qquad \dot{q} = \beta u/\alpha - \Delta\dot{\phi}/\alpha. \tag{6}$$

For $|u| < (\gamma_0 H_{\rm K}/2)\Delta(\pi/4)\alpha/|\beta-\alpha|$ [28], a stationary regime exists with $\dot{q} = \beta u/\alpha$. The DW velocity is remarkably independent of the DW width, as already shown with (4). Above threshold ϕ oscillates and so does the velocity. The average wall velocity, plotted in fig. 3, is very similar in behaviour to numerical results (fig. 2a). The non-trivial effect of roughness is not included in this 1D model, but if one takes it into account by an increased dissipation (larger α) a velocity reduction in rough wires is natural.

We finally turn to discussing possible origins of eq. (3). One may first remark that the new term exhausts the possibilities of introducing linearly the magnetization gradient along the electric current direction into the LLG equation.

The approach of Tatara and Kohno [13] put forward two effects of an electric current across a DW, called spin transfer and momentum transfer. The former, dominant for thick DWs, has the same form as our first term, while the latter, dominant for thin walls, enters the integrated DW dynamics equations (6) similarly to our second term. Thus, for a DW of finite thickness the second term should remain present in some proportion. As the characteristic length is here the Fermi wavelength [13], we expect $\beta \ll 1$. Note however the lack of an expression in the intermediate DW thickness regime that would predict the value of β.

Zhang *et al.* calculated spin accumulation for a current flowing across a multilayer, and the resultant torque on the magnetization, with the inclusion of the *s*-*d* exchange interaction [11, 29]. Extending their model [11] to the case of a linearly varying magnetization gives both terms of (3) with a slightly modified equivalent velocity u and a β factor given by

$$\beta = (\lambda_J/\lambda_{\rm sf})^2 = \hbar/\left(J\tau_{\rm sf}\right), \tag{7}$$

with J the *s*-*d* exchange interaction energy, $\tau_{\rm sf}$ the spin-flip time, λ_J and $\lambda_{\rm sf}$ the associated diffusion lengths. Note that with $\lambda_J = 1\,$nm and $\lambda_{\rm sf} = 5\,$nm for permalloy, one gets $\beta = 0.04$. As for pure metals $\lambda_{\rm sf}$ can be much larger [30], one expects that β may become smaller and, hence, that DWs may prove more difficult to move. This should be investigated by experiments. One should also be aware that the time $\tau_{\rm sf}$ in (7) may not be exactly the spin-flip time. This time appears in fact as the transverse relaxation time in the evolution equation of the spin accumulation. If part of this relaxation occurs through interaction with the localized moments, for example by incoherent precession around the local magnetization, then the moment is transferred to the localized spin system and does not contribute to β.

In conclusion, we have shown that the inclusion of a second term describing the spin transfer torque, akin to the exchange field term considered previously, might solve the puzzle of the quantitative explanation by micromagnetics of the current-induced DW motion [31]. This view is supported by a direct comparison of experimental results (DW velocity, reduction of DW propagation field under current) to micromagnetic simulations of realistic nanowires (excepting pinning phenomena). The model predicts that DWs should move under smaller currents for more perfect wires (having a lower propagation field). As our approach was phenomenological, a basic understanding of the form of the spin-transfer torque in continuous structures as wide as a domain wall is still very much needed. The magnitude of the new term, and its material dependence, should allow to discriminate between models.

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